2D Volume Integral Formulations for Nonlinear Magneto-static Field Computation for Rotating Machines Pre-Design Considering Periodicities

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***Abstract*—This paper provides an alternative approach to the finite elements method for solving magneto-static field for an efficient pre-design of electrical rotating machines [1]-[2]. At first, the 2D volume integral formulation will be presented. Secondly, this formulation will be extended to periodic systems.**

***Index Terms* —2D integral method, magneto static, electrical engine pre-design, field computation**

I. 2D Volume Integral method

The Volume Integral Method (VIM) has been developed in 3D in [3] and lightly presented in [4]. This method relies on the vector magnetic potential $\mathbf{A}$, defined as $\mathbf{B} = \nabla \times \mathbf{A}$ interpolated on nodal elements as $\mathbf{A} = \sum_{i=1}^{Nodes} \mathbf{w}_i A_i$, $\mathbf{w}_i$ being the first order shape function of nodal elements. In 2D, the operator "$\nabla \times \mathbf{w}$" corresponds to "$\nabla \mathbf{w} \times \mathbf{n}_p$" where $\mathbf{n}_p$ is the normal to the plane under review. On one hand, we have $\mathbf{H} = \nu \mathbf{B} - \mathbf{H}_c$ ($\mathbf{H}_c$ is the permanent magnet field and ν the reluctivity of the material). The Maxwell-Ampère equation $\nabla \times \mathbf{H} = \mathbf{J}_0$ allows to express the magnetic field as :

$$\mathbf{H} = \mathbf{H}_0 - \nabla \varphi_r \tag{1}$$

where $\mathbf{H}_0$ is the source term created by $\mathbf{J}_0$ in the vacuum. The scalar potential φ_r can be expressed as a function of the magnetization $\mathbf{M}$:

$$\varphi_r = \frac{1}{2\pi} \int_{\Omega_m} \mathbf{M} \cdot \nabla \ln(r) \tag{2}$$

where is defined by the relation $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$, r the distance between the source and the target and Ω_m the ferromagnetic domain. From those relations, the following system can be obtained [3] :

$$(\mathbf{R} + \mathbf{L})\mathrm{A} = \mathrm{U}_0 \tag{3}$$

in which matrices $\mathbf{R}$, $\mathbf{L}$ and vector U_0 are defined as :

$$\mathbf{R}_{\mathrm{ij}} = \int_{\Omega_m} \nabla \times \mathbf{w}_i \cdot \nu \nabla \times \mathbf{w}_j$$

$$\mathrm{U}_{0\mathrm{i}} = \int_{\Omega_m} \nabla \times \mathbf{w}_i \cdot (\mathbf{H}_0 + \mathbf{H}_c) + \frac{1}{2\pi} \int_{\Gamma_{\mathrm{ext}}} (\nabla \times \mathbf{w}_i \cdot \mathbf{n}_\Gamma) \int_{\Omega_m} \mathbf{H}_c \cdot \nabla \ln(r) \tag{4}$$

$$\mathbf{L}_{\mathrm{ij}} = \frac{1}{2\pi} \int_{\Gamma_{ext}} (\nabla \times \mathbf{w}_i \cdot \mathbf{n}_\Gamma) \int_{\Omega_m} (\nu - \nu_0) \nabla \times \mathbf{w}_j \cdot \nabla \ln(r)$$

II. Validation

The formulation has been tested by calculating the field in the air gap of a permanent magnet rotating machine (PMRM). Since this machine is periodic, only one quarter is shown in Fig.1. The radial magnets are opposed in direction. The coils of PMRM are powered with a 1000A tri-phased current.

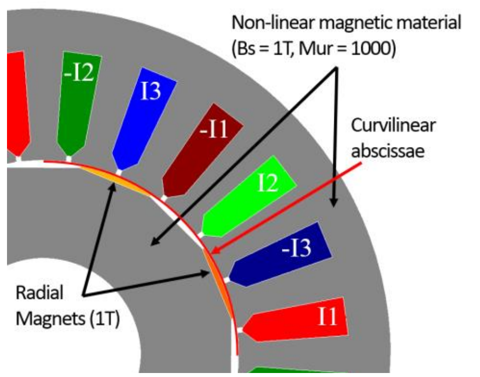

Fig. 1 Description of test case n°1

The magnetic field in the air gap of the machine is computed with the relation (5). The results obtained with the VIM are compared to a reference finite element solution and plotted in Fig. 2.

$$B_{airGap} = \frac{\mu_0}{2\pi} \nabla \int_{\Omega_m} \mathbf{M} \cdot \nabla \ln(r) + \mu_0 \mathbf{H}_0 \tag{5}$$

For this test case, we used a quite poor quality mesh (900 unknowns for the full PMRM) that nevertheless gave us a mean error of less than 5%. We can conclude from this test that even with few degrees of freedom (DOF), the VIM is efficient for difficult cases of magneto static field computation.

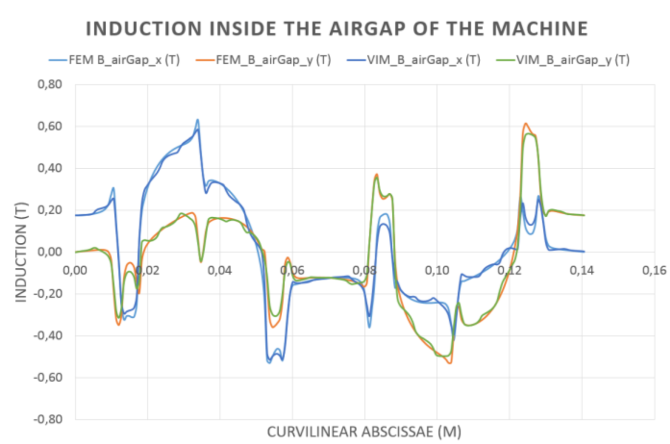

Fig. 2 Induction in the airgap of the rotation machine

III. Periodicity Adapted Formulation

Since most PMRM are periodic one would want to exploit this characteristic in order to decrease the number of unknowns and thus the computation time. An adapted formulation for the VIM is described below. Fig. 3 is a simplified example to illustrate this periodic volume integral method (PVIM). The DOF used in this periodic formulations are the ones contained in the "Real" domain (right).

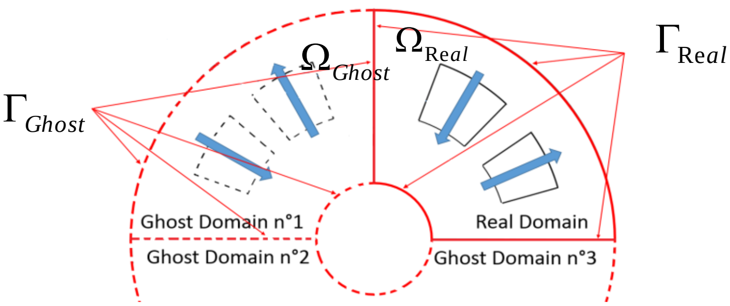

Fig. 3 Detailed geometry for the periodic formulation

The "Ghost" domains (left and below) are seen as "sources terms" for the distance interactions, thus only the full matrix and the second hand are modified and the FEM like matrix $\mathbf{R}$ stays the same. The boundary conditions are treated by the integral matrix $\mathbf{L}$. Parameter a_{Ghost} is used extend the periodic formulation to an antiperiodic formulation ($a_{Ghost} = \pm 1$ when the ghost domains are periodic (+) / antiperiodic (-)). $H_{c_{Ghost}}$ is the field of the permanent magnets in the ghost domains:

$$
\begin{aligned}
\mathrm{U}_{0\mathrm{i}} &= \int_{\Omega_m} \nabla\times\mathbf{w}_i\cdot\left(H_0+H_c\right)+\frac{1}{2\pi}\int_{\Gamma_{\mathrm{Real}}}(\nabla\times\mathbf{w}_i\cdot n)\int_{\Omega_{\mathrm{Real}}} H_c\cdot\nabla\ln(r) \\
&+\frac{1}{2\pi}\sum_{NbGhost} a_{Ghost}\left(\int_{\Gamma_{\mathrm{Ghost}}}(\nabla\times\mathbf{w}_i\cdot n)\int_{\Omega_{\mathrm{Real}}} H_{c_{Ghost}}\cdot\nabla\ln(r)\right) \\
\mathbf{L}_{\mathrm{ij}} &= \frac{1}{2\pi}\int_{\Gamma_{\mathrm{Real}}}(\nabla\times\mathbf{w}_i\cdot n)\int_{\Omega_{\mathrm{Real}}}(\nu-\nu_0)\nabla\times\mathbf{w}_j\cdot\nabla\ln(r)+ \\
&\frac{1}{2\pi}\sum_{NbGhost} a_{Ghost}\left(\int_{\Gamma_{Ghost}}(\nabla\times\mathbf{w}_i\cdot n)\int_{\Omega_{\mathrm{Real}}}(\nu-\nu_0)\nabla\times\mathbf{w}_j\cdot\nabla\ln(r)\right)
\end{aligned} \tag{6}
$$

Thanks to the antiperiodic extension, the real domain becomes one eighth of the full PRRM. It is still possible to decrease again the number of unknowns since one eighth of the domain presents an axial symmetry (Fig. 4). The combination of periodicity and symmetry is nevertheless only for specific cases since the symmetry is easily lost (movement of the rotor or non-symmetric current sources).

IV. Computation Performance

To test this new formulation we will calculate the induction inside the airgap of a PMRM in open circuit. This configuration is used to compute the cogging torque and matches the requirements of the symmetric case. The results computation times of this study are summed up in the Tab. 1 (computation ran on a Intel® Core™ i7-2720QM 2.20Ghz CPU).

	Linear system construction	1 iteration of NR solver	Total computation time
Full geometry	3.75s/0.41s	2.02s/0.40s	33.78s/5.16s
1/4 geometry (Gmres solver)	0.94s/0.12s	0.53s/0.081s	8.22s/1.37s
1/4 geometry (direct solver)	0.85s/0.29s	0.46s/0.044s	7.55s/0.89s
1/8th geometry (Gmres solver)	0.58s/0.19s	0.27s/0.081s	4.27s/1.12s
1/8th geometry (direct solver)	0.72s/0.16s	0.13s/0.023s	2.80s/0.45s
1/16th geometry (Gmres solver)	0.34s/0.16s	0.18s/0.034s	2.49s/0.68s
1/16th geometry (direct solver)	0.48s/0.057s	0.051s/0.015s	1.24s/0.22s

Tab. 1 Comparison of computation time for both formulations and solvers

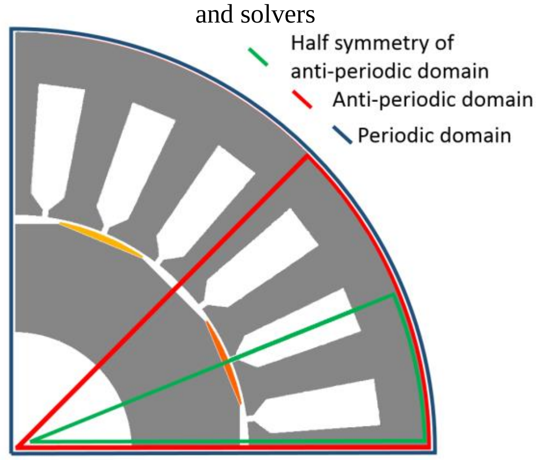

Fig. 4 Periodicity, anti-periodicity and symmetry of test case

The values on the left (red) are calculated with a fine mesh (3600 nodes for the full geometry) while the values on the right (blue) are calculated with a poor quality mesh (900 nodes for the full geometry). As expected, taking advantage of the periodicity/anti-periodicity/symmetry in the formulation decrease significantly the computation time.

Conclusion

An original integral formulation has been developed and put into application. We have seen that it is adapted to rotating machine pre-design since it yields good quality results with few unknowns. Taking advantages of the periodicities allows as well to highly reduce the computation time.

References

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